MULTIMEDIA UNIVERSITY

STUDENT ID NO

SEAT NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2022/2023

DMA5401 – INTRODUCTION TO PROBABILITY AND STATISTICS

(All sections / Groups)

3 OCTOBER 2023
9.00am – 11.00am
(2 Hours)

Question #	Marks
1	
2	
3	
4	
Total	

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **13** pages with **4** questions only excluding the cover page.

2. Attempt **ALL** questions. All necessary working steps must be shown.

3. Please write your answers in the **QUESTION BOOKLET**. It is your responsibility to make sure that this paper is submitted at the end of the examination.

QUESTION 1 [10 MARKS]

(a) Siti sells muffins to increase her personal income and the probability of making a sale is 0.1. Given that she randomly approaches 30 people, find the probability that she will make at least 2 sales. [3 marks]

(b) A personnel test is designed to test a job applicant's cognitive and physical abilities. It is known that for all the tests administrated last year, the distribution of scores was approximately normal with a mean of 75 and a standard deviation of 7.5. Find the value of h if $P(X < h) = 0.9$. Round up your answer to an integer.

[3 marks]

Continued...

(c) The number of traffic accidents that occur on a particular stretch of road during a month (30 days) follows a Poisson distribution with a mean of 4. Find the probability that

(i) less than two accidents will occur next month on this stretch of road.

[2 marks]

(ii) at least one accident will occur in the next 15 days on this stretch of road.

[2 marks]

Continued...

QUESTION 2 [10 MARKS]

(a) Suppose we wish to estimate the average execution time of a program. The program was run six times with randomly chosen data sets, and the sample mean of the execution times was evaluated as 240 seconds and the sample standard deviation as 14 seconds. Construct a 95% confidence interval of the mean execution time.

[3 marks]

(b) Suppose that we wish to estimate the true mean of the length of wooden rod produced in a factory, and that we want to be at least 95% confidence that the error is at most 2. How large a sample will we need if we know that $\sigma = 15$?

[3 marks]

Continued...

(c) ABC Corporation makes auto batteries. From a random sample of 150 such batteries selected, 115 of the batteries can last for at least 2 years.

(i) Estimate the sample proportion of batteries that can last for at least 2 years.

(ii) Construct a 90% confidence interval for the proportion of all batteries that can last for at least 2 years.

[4 marks]

Continued...

QUESTION 3 [10 MARKS]

(a) A manufacturer of snack food claims that their food package has an average weight of 130g. A random sample of 9 packages, when tested, yields a sample average weight of 131g. If the distribution of package weight is normal with population standard deviation of 1.5g, does the data contradict the manufacturer's claim at significance level $\alpha = 0.01$?

[5 marks]

Continued...

(b) Suppose that in an experiment to determine whether the use of robots to weave computer cables is feasible, a robot was used to assemble 500 cables. The cables were examined and there were 14 defectives. If human assemblers have a defect rate of 0.03 (3%), does this data support the hypothesis that the proportion of defectives is lower for robots than humans? Use a 0.01 significance level.

[5 marks]

Continued...

QUESTION 4 [10 MARKS]

(Note: Use 3 decimal places in your calculation.)

The scores for a sample of 8 students on the DMA5401 midterm test (x) and final examination (y) are as follows.

x	76	49	70	71	80	83	96	99
y	75	43	68	72	79	84	97	96

(a) (i) Compute $\sum x$, $\sum y$, $\sum xy$, $\sum x^2$, S_{xx}, S_{xy},

(ii) Compute b_1, b_0

(iii) Find the equation of the regression line.

[6 marks]

Continued...

(b) Benny scored 75 on the midterm test but was ill at the time of the final examination. Estimate his final examination score. [1 mark]

(c) Calculate the correlation coefficient between x and y. Explain your finding. [3 marks]

Continued...

Appendix

Binomial Distribution

$$P(X=x)={}^{n}C_{x}p^{x}(1-p)^{n-x}$$

Poisson Distribution

$$P(X=x)=\frac{e^{-\lambda}\lambda^{x}}{x!}$$

Normal Distribution

$$Z=\frac{X-\mu}{\sigma}$$

Estimation

$(1-\alpha)100\%$ confidence interval for $\mu=\bar{x}\pm Z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right)$

$(1-\alpha)100\%$ confidence interval for $\mu=\bar{x}\pm t_{\frac{\alpha}{2},\nu=n-1}\left(\frac{s}{\sqrt{n}}\right)$

$(1-\alpha)100\%$ confidence interval for $p=\hat{p}\pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

Determine Sample size

$$n=\left(\frac{Z_{\alpha/2}\sigma}{E}\right)^{2}$$

$$n=\left(\frac{Z_{\alpha/2}\sqrt{\hat{p}\hat{q}}}{E}\right)^{2}$$

Hypothesis Testing

Test statistics: $Z=\frac{\bar{x}-\mu}{\sigma_{\bar{x}}}$, $\sigma_{\bar{x}}=\frac{\sigma}{\sqrt{n}}$

Test statistics: $t=\frac{\bar{x}-\mu}{s_{\bar{x}}}$; $s_{\bar{x}}=\frac{s}{\sqrt{n}}$

Test statistics: $Z=\frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}}$

Continued...

Regression and Correlation

$$S_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n}$$

$$S_{xy} = \sum xy - \frac{\left(\sum x\right)\left(\sum y\right)}{n}$$

$$S_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n}$$

$$b_1 = \frac{S_{XY}}{S_{XX}}$$

$$b_0 = \bar{y} - b_1\bar{x} \quad ; \bar{y} = \frac{\sum y}{n} \quad ; \bar{x} = \frac{\sum x}{n}$$

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$$

Continued...

Standard Normal Distribution

The Normal Distribution Function

$$p(z \le z_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-\frac{1}{2}z^2} dz$$

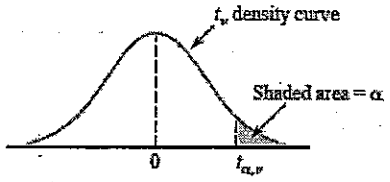
x	Φ(x)	x	Φ(x)	x	Φ(x)	x	Φ(x)
0.00	0.5000	0.50	0.6915	1.00	0.8413	1.50	0.9332
0.01	0.5040	0.51	0.6950	1.01	0.8438	1.51	0.9345
0.02	0.5080	0.52	0.6985	1.02	0.8461	1.52	0.9357
0.03	0.5120	0.53	0.7019	1.03	0.8485	1.53	0.9370
0.04	0.5160	0.54	0.7054	1.04	0.8508	1.54	0.9382
0.05	0.5199	0.55	0.7088	1.05	0.8531	1.55	0.9394
0.06	0.5239	0.56	0.7123	1.06	0.8554	1.56	0.9406
0.07	0.5279	0.57	0.7157	1.07	0.8577	1.57	0.9418
0.08	0.5319	0.58	0.7190	1.08	0.8599	1.58	0.9429
0.09	0.5359	0.59	0.7224	1.09	0.8621	1.59	0.9441
0.10	0.5398	0.60	0.7257	1.10	0.8643	1.60	0.9452
0.11	0.5438	0.61	0.7291	1.11	0.8665	1.61	0.9463
0.12	0.5478	0.62	0.7324	1.12	0.8686	1.62	0.9474
0.13	0.5517	0.63	0.7357	1.13	0.8708	1.63	0.9484
0.14	0.5557	0.64	0.7389	1.14	0.8729	1.64	0.9495
0.15	0.5596	0.65	0.7422	1.15	0.8749	1.65	0.9505
0.16	0.5636	0.66	0.7454	1.16	0.8770	1.66	0.9515
0.17	0.5675	0.67	0.7486	1.17	0.8790	1.67	0.9525
0.18	0.5714	0.68	0.7517	1.18	0.8810	1.68	0.9535
0.19	0.5753	0.69	0.7549	1.19	0.8830	1.69	0.9545
0.20	0.5793	0.70	0.7580	1.20	0.8849	1.70	0.9554
0.21	0.5832	0.71	0.7611	1.21	0.8869	1.71	0.9564
0.22	0.5871	0.72	0.7642	1.22	0.8888	1.72	0.9573
0.23	0.5910	0.73	0.7673	1.23	0.8907	1.73	0.9582
0.24	0.5948	0.74	0.7704	1.24	0.8925	1.74	0.9591
0.25	0.5987	0.75	0.7734	1.25	0.8944	1.75	0.9599
0.26	0.6026	0.76	0.7764	1.26	0.8962	1.76	0.9608
0.27	0.6064	0.77	0.7794	1.27	0.8980	1.77	0.9616
0.28	0.6103	0.78	0.7823	1.28	0.8997	1.78	0.9625
0.29	0.6141	0.79	0.7852	1.29	0.9015	1.79	0.9633
0.30	0.6179	0.80	0.7881	1.30	0.9032	1.80	0.9641
0.31	0.6217	0.81	0.7910	1.31	0.9049	1.81	0.9649
0.32	0.6255	0.82	0.7939	1.32	0.9066	1.82	0.9656
0.33	0.6293	0.83	0.7967	1.33	0.9082	1.83	0.9664
0.34	0.6331	0.84	0.7995	1.34	0.9099	1.84	0.9671
0.35	0.6368	0.85	0.8023	1.35	0.9115	1.85	0.9678
0.36	0.6406	0.86	0.8051	1.36	0.9131	1.86	0.9686
0.37	0.6443	0.87	0.8078	1.37	0.9147	1.87	0.9693
0.38	0.6480	0.88	0.8106	1.38	0.9162	1.88	0.9699
0.39	0.6517	0.89	0.8133	1.39	0.9177	1.89	0.9706
0.40	0.6554	0.90	0.8159	1.40	0.9192	1.90	0.9713
0.41	0.6591	0.91	0.8186	1.41	0.9207	1.91	0.9719
0.42	0.6628	0.92	0.8212	1.42	0.9222	1.92	0.9726
0.43	0.6664	0.93	0.8238	1.43	0.9236	1.93	0.9732
0.44	0.6700	0.94	0.8264	1.44	0.9251	1.94	0.9738
0.45	0.6736	0.95	0.8289	1.45	0.9265	1.95	0.9744
0.46	0.6772	0.96	0.8315	1.46	0.9279	1.96	0.9750
0.47	0.6808	0.97	0.8340	1.47	0.9292	1.97	0.9756
0.48	0.6844	0.98	0.8365	1.48	0.9306	1.98	0.9761
0.49	0.6879	0.99	0.8389	1.49	0.9319	1.99	0.9767
0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.9772

Continued...

x	Φ(x)	x	Φ(x)	x	Φ(x)	x	Φ(x)
2.00	0.97725	2.50	0.99379	3.00	0.99865	3.50	0.99977
2.01	0.97778	2.51	0.99396	3.01	0.99869	3.51	0.99978
2.02	0.97831	2.52	0.99413	3.02	0.99874	3.52	0.99978
2.03	0.97882	2.53	0.99430	3.03	0.99878	3.53	0.99979
2.04	0.97932	2.54	0.99446	3.04	0.99882	3.54	0.99980
2.05	0.97982	2.55	0.99461	3.05	0.99886	3.55	0.99981
2.06	0.98030	2.56	0.99477	3.06	0.99889	3.56	0.99981
2.07	0.98077	2.57	0.99492	3.07	0.99893	3.57	0.99982
2.08	0.98124	2.58	0.99506	3.08	0.99896	3.58	0.99983
2.09	0.98169	2.59	0.99520	3.09	0.99900	3.59	0.99983
2.10	0.98214	2.60	0.99534	3.10	0.99903	3.60	0.99984
2.11	0.98257	2.61	0.99547	3.11	0.99906	3.61	0.99985
2.12	0.98300	2.62	0.99560	3.12	0.99910	3.62	0.99985
2.13	0.98341	2.63	0.99573	3.13	0.99913	3.63	0.99986
2.14	0.98382	2.64	0.99585	3.14	0.99916	3.64	0.99986
2.15	0.98422	2.65	0.99598	3.15	0.99918	3.65	0.99987
2.16	0.98461	2.66	0.99609	3.16	0.99921	3.66	0.99987
2.17	0.98500	2.67	0.99621	3.17	0.99924	3.67	0.99988
2.18	0.98537	2.68	0.99632	3.18	0.99926	3.68	0.99988
2.19	0.98574	2.69	0.99643	3.19	0.99929	3.69	0.99989
2.20	0.98610	2.70	0.99653	3.20	0.99931	3.70	0.99989
2.21	0.98645	2.71	0.99664	3.21	0.99934	3.71	0.99990
2.22	0.98679	2.72	0.99674	3.22	0.99936	3.72	0.99990
2.23	0.98713	2.73	0.99683	3.23	0.99938	3.73	0.99990
2.24	0.98745	2.74	0.99693	3.24	0.99940	3.74	0.99991
2.25	0.98778	2.75	0.99702	3.25	0.99942	3.75	0.99991
2.26	0.98809	2.76	0.99711	3.26	0.99944	3.76	0.99992
2.27	0.98840	2.77	0.99720	3.27	0.99946	3.77	0.99992
2.28	0.98870	2.78	0.99728	3.28	0.99948	3.78	0.99992
2.29	0.98899	2.79	0.99736	3.29	0.99950	3.79	0.99992
2.30	0.98928	2.80	0.99744	3.30	0.99952	3.80	0.99993
2.31	0.98956	2.81	0.99752	3.31	0.99953	3.81	0.99993
2.32	0.98983	2.82	0.99760	3.32	0.99955	3.82	0.99993
2.33	0.99010	2.83	0.99767	3.33	0.99957	3.83	0.99994
2.34	0.99036	2.84	0.99774	3.34	0.99958	3.84	0.99994
2.35	0.99061	2.85	0.99781	3.35	0.99960	3.85	0.99994
2.36	0.99086	2.86	0.99788	3.36	0.99961	3.86	0.99994
2.37	0.99111	2.87	0.99795	3.37	0.99962	3.87	0.99995
2.38	0.99134	2.88	0.99801	3.38	0.99964	3.88	0.99995
2.39	0.99158	2.89	0.99807	3.39	0.99965	3.89	0.99995
2.40	0.99180	2.90	0.99813	3.40	0.99966	3.90	0.99995
2.41	0.99202	2.91	0.99819	3.41	0.99968	3.91	0.99995
2.42	0.99224	2.92	0.99825	3.42	0.99969	3.92	0.99996
2.43	0.99245	2.93	0.99831	3.43	0.99970	3.93	0.99996
2.44	0.99266	2.94	0.99836	3.44	0.99971	3.94	0.99996
2.45	0.99286	2.95	0.99841	3.45	0.99972	3.95	0.99996
2.46	0.99305	2.96	0.99846	3.46	0.99973	3.96	0.99996
2.47	0.99324	2.97	0.99851	3.47	0.99974	3.97	0.99996
2.48	0.99343	2.98	0.99856	3.48	0.99975	3.98	0.99997
2.49	0.99361	2.99	0.99861	3.49	0.99976	3.99	0.99997
2.50	0.99379	3.00	0.99865	3.50	0.99977	4.00	0.99997

Continued...

Table A.5 Critical Values for t Distributions



	α						
ν	.10	.05	.025	.01	.005	.001	.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	1.299	1.676	2.009	2.403	2.678	3.262	3.496
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

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